ON A TYPE OF SUBGROUPS OF A COMPACT LIE GROUP

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Let G be a connected compact Lie group and H a connected closed subgroup. Then H is an orientable submanifold of G and we may consider H as a cycle in G. In his interesting paper on the topology of group manifolds[1] H. Samelson has proved that, if H is not homologous to 0, then the homology ring[2] of the coset space G/H is isomorphic to the homology ring of a product space of odd dimensional spheres and the homology ring of G is isomorphic to that of the product of the spaces H and G/H. On the other hand, in a recent investigation of fibre bundles[3] T. Kudo has shown that, if the homology ring of the coset space G/H is isomorphic to that of an odd dimensional sphere, then H is not homologous to 0.

In the present paper we shall consider those connected closed subgroups of a connected compact Lie group G such that the homology rings of the coset spaces are isomorphic to that of odd dimensional spheres. We shall first show that the problem to find all such subgroups of G may be reduced to the case where G is a simple group. The determination of such subgroups of the rotation groups of spheres (simple Lie groups of types B and D) is contained essentially in a paper by D. Montgomery and H. Samelson on the transformation groups of spheres.[4] Hence we shall consider here the above problem for simple Lie groups of the other types. The writer is grateful to Mr. M. Kuranishi for his friendly cooperation during the preparation of this paper.

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1. All groups considered in the following are compact Lie groups and sub-

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[1] H. Samelson, Beiträge zur Topologie der Gruppen-Mannigfaltigkeiten, Ann. of Math. Vol. **42** (1941); Satz VI. We refer to this paper as [S].

[2] The coefficients of the homology ring are rational numbers.

[3] T. Kudo, On the homological properties of fibre bundles, forthcoming in Journ. of the Institute of Polytechnics, Osaka City University.

[4] D. Montgomery and H. Samelson, Transformation groups of spheres, Ann. of Math. Vol. **44** (1943). We refer to this paper as [M-S].