ON THE STRUCTURE OF COMPLETE LOCAL RINGS

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The concept of a local ring was introduced by Krull [2],[0] who defined it as a Noetherian ring R (we say that a commutative ring R is Noetherian if every ideal in R has a finite basis and if R contains the identity) which has only one maximal ideal $\mathfrak{m}$. If the powers of $\mathfrak{m}$ are defined as a system of neighbourhoods of zero, then R becomes a topological ring satisfying the first axiom of countability. And the notion was studied recently by C. Chevalley and I. S. Cohen. Cohen [1] proved the structure theorem for complete rings besides other properties of local rings.

The main purpose of the present paper is to show that the structure theorem holds for rings satisfying somewhat weaker condition; for local rings in the sense of this paper (cf. Definition 1).

Appendix (1) shows some other properties of local rings; they may be considered as generalizations of Lemma 1 and Theorems 4, 7 and 8 in Part I, [1]. Further, Appendix (2) is to show an example of non-Noetherian local ring whose maximal ideal has a finite basis (non-Noetherian generalized local ring in the sense of Cohen [1]).

As for terminology, a ring means, throughout this paper, a commutative ring with identity. Under a subring we mean a subring having the same identity element.

DEFINITION 1. A local ring R is a ring in which (1) the set $\mathfrak{m}$ of non-units form an ideal and (2) $\bigcap_{n=1}^{\infty} \mathfrak{m}^n = (0)$.

In any local ring R a topology can be introduced by taking ideals $\mathfrak{m}, \mathfrak{m}^2, \ldots$ to be neighbourhoods of zero. This is the natural topology of a local ring.

DEFINITION 2. An absolutely unramified local ring is a local ring with the maximal ideal (p) where p is zero or a prime number.

DEFINITION 3. If R and R' are two local rings such as (1) R is a subring of R' and (2) non-units in R are non-units in R', then we say that R is a special subring of R'.

LEMMA 1. Any local ring contains at least one absolutely unramified local

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[0] The number in brackets refers to the bibliography at the end.