CONSTRUCTION AND CHARACTERIZATION OF GALOIS ALGEBRAS WITH GIVEN GALOIS GROUP

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Recently H. Hasse[1] has given an interesting theory of Galois algebras, which generalizes the well known theory of Kummer fields; an algebra $\mathfrak{A}$ over a field Ω is called a Galois algebra with Galois group G when $\mathfrak{A}$ possesses G as a group of automorphisms and $\mathfrak{A}$ is (G, Ω)-operator-isomorphic to the group ring $G(\Omega)$ of G over Ω.[2] On assuming that the characteristic of Ω does not divide the order of G and that absolutely irreducible representations of G lie in Ω, Hasse constructs certain Ω-basis of $\mathfrak{A}$, called factor basis, in accord with Wedderburn decomposition of the group ring and shows that a characterization of $\mathfrak{A}$ is given by a certain matrix factor system which defines the multiplication between different parts of the factor basis belonging to different characters of G. Now the present work is to free the theory from the restriction on the characteristic. We can indeed embrace the case of non-semisimple modular group ring $G(\Omega)$.

1. Decomposition of group ring.[3] Let G be a finite group whose absolutely irreducible representations lie in a field Ω. Let $\mathfrak{G} = G(\Omega)$ be its group ring over Ω. Let

$$1 = \sum_{\kappa=1}^{k} \sum_{i=1}^{f(\kappa)} e_i^{(\kappa)} \tag{1}$$

be a decomposition of 1 into a sum of mutually orthogonal primitive idempotent elements in $\mathfrak{G}$, where the left-(or, right-)ideals generated by $e_1^{(\kappa)}, \ldots, e_{f(\kappa)}^{(\kappa)}$ are isomorphic while those generated by $e_i^{(\kappa)}$, $e_j^{(\lambda)}$ with $\kappa \neq \lambda$ are not. Let $c_{ij}^{(\kappa)}$ be, for each κ, a corresponding system of matric units. For simplicity's sake we denote $e_1^{(\kappa)}$ by $e^{(\kappa)}$. Let

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1) [2].

2) Hasse demands further that $\mathfrak{A}$ be associative, commutative and, moreover, semisimple.

3) Cf. e.g. [3].