

A REMARK ON THE THEOREM OF OHSAWA-TAKEGOSHI

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§1. Introduction and main result

If $D \subset \mathbb{C}^n$ is a pseudoconvex domain and $X \subset D$ a closed analytic subset, the famous theorem B of Cartan-Serre asserts, that the restriction operator $r : \mathcal{O}(D) \longrightarrow \mathcal{O}(X)$ mapping each function F to its restriction $F|_X$ is surjective. A very important question of modern complex analysis is to ask what happens to this result if certain growth conditions for the holomorphic functions on D and on X are added. If the L^2 -norm with respect to the Lebesgue-measure and a plurisubharmonic weight function is taken as growth condition, then the Cartan-Serre extension has the following analogue:

THEOREM 1.1. (Ohsawa-Takegoshi [3]) *Let $D \subset \subset \mathbb{C}^n$ be a bounded pseudoconvex domain, $H \subset \mathbb{C}^n$ a complex affine hyperplane with $D' := D \cap H \neq \emptyset$ and $\varphi : D \longrightarrow \mathbb{R} \cup \{-\infty\}$ a plurisubharmonic function. Then there is a constant $C > 0$, depending only on the diameter of D , such that for each function f holomorphic on D' satisfying the growth condition*

$$\int_{D'} |f|^2 e^{-\varphi} dV_{n-1} < \infty,$$

where dV_{n-1} denotes the Lebesgue-measure on $X \cong \mathbb{R}^{2n-2}$ there is a holomorphic function F on D such that $r(F) = F|_{D'} = f$ and

$$\int_D |F|^2 e^{-\varphi} dV_n \leq C \int_{D'} |f|^2 e^{-\varphi} dV_{n-1}.$$

This theorem has, meanwhile, found a lot of applications in complex analysis and in algebraic geometry. Therefore, it is important to ask what kind of generalizations are possible. We mention the work of L. Manivel [2]

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