A NOTE ON TANGENT BUNDLES

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Dedicated to Professor K. Noshiro for his 60th birthday

The tangent bundle of a differentiable manifold is an important invariant of a differentiable structure. It is determined neither by the topological structure nor by the homotopy type of a manifold. But in some cases tangent bundles depend only on the homotopy types of manifolds.

In this note we shall show that homotopy spheres and homotopy real projective spaces have homotopically equivalent tangent bundles respectively. Also, the action of Θ_n, the group of the homotopy spheres, on an oriented smooth manifold by the connected sum does not have an effect on the structure of the tangent bundle ($n \geqq 5$).

1. Let M^n be a differentiable manifold of dimension n. Let ξ, ξ' be vector bundles over M^n. If ξ is equivalent (or isomorphic) to ξ', then we shall denote it by $\xi \approx \xi'$. Let $\tau(M^n)$ be the tangent bundle of M^n.

Theorem 1. *Let $\sum^n$ be a homotopy n-sphere. Let $f: S^n \to \sum^n$ be an orientation preserving homotopy equivalence of the standard n-sphere S^n onto $\sum^n$. Then $f^*(\tau(\sum^n)) \approx \tau(S^n)$. In other words, f is covered by a bundle map $\tilde{f}$ of $\tau(S^n)$ onto $\tau(\sum^n)$.*

Remark. If n is even and $n \not\equiv 2 \pmod 8$, then this is a consequence of a theorem of Takeuchi [11].

Proof. If $n \leqq 7$, Theorem 1 is well known and derived by the similar argument that follows. Therefore, we assume $n \geqq 8$. Let $\tau' = f^*\tau(\sum^n)$ and $\tau = \tau(S^n)$. We shall show that $\tau \approx \tau'$. Let ξ be an oriented n-plane bundle over S^n. Let $\alpha(\xi) \in \pi_{n-1}(SO(n))$ be the characteristic class of ξ. By the classification theorem (Steenrod [10]) it is sufficient to prove that $\alpha(\tau) = \alpha(\tau')$. Let $i: SO(n) \to SO(n+1)$ be the inclusion, and let $i_*: \pi_{n-1}(SO(n)) \to \pi_{n-1}(SO(n+1))$ be the induced

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