ON THE BALAYAGE FOR LOGARITHMIC POTENTIALS

NOBUYUKI NINOMIYA

To Professor KIYOSHI NOSHIRO on the occasion of his 60th birthday

In this paper, we shall consider the logarithmic potential

$$U^{\mu}(P)=\int \log \frac{1}{PQ}\, d\mu(Q),$$

where μ is a positive measure in the plane, P and Q are any points and PQ denotes the distance from P to Q. In general, consider the potential

$$K(P,\ \mu)=\int K(P,\ Q)\, d\mu(Q)$$

of a positive measure μ taken with respect to a kernel $K(P,\ Q)$ which is a continuous function in P and Q and may be $+\infty$ for $P=Q$. A kernel $K(P,\ Q)$ is said to satisfy the balayage principle if, given any compact set F and any positive measure μ with compact support, there exists a positive measure μ' supported by F such that $K(P,\ \mu')=K(P,\ \mu)$ on F with a possible exception of a set of K-capacity zero and $K(P,\ \mu')\leqq K(P,\ \mu)$ everywhere. A kernel $K(P,\ Q)$ is said to satisfy the equilibrium principle if, given any compact set F, there exists a positive measure λ supported by F such that $K(P,\ \lambda)=V$ (a constant) on F with a possible exception of a set of K-capacity zero and $K(p,\ \lambda)\leqq V$ everywhere. The logarithmic kernel

$$K(P,\ Q)=\log \frac{1}{PQ}$$

satisfies the equilibrium principle in the plane, but it does not satisfy the balayage principle in the above form. As is well-known, given any compact set F and any point M of the complement CF of F, there exist a positive measure ε' supported by F with total mass 1 and a non-negative constant γ such that

(1) $U^{\varepsilon'}(P)=\log \dfrac{1}{MP}+\gamma$ on F with a possible exception of a set of logarithmic capacity zero, and

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