A METHOD OF TWO-LEVEL SIMPLIFICATION OF BOOLEAN FUNCTIONS

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Dedicated to Professor K. NOSHIRO on his sixtieth birthday

There are a number of methods to find minimal two-level forms for a given Boolean function, e g. Harvard's group [1], Veitch [2], Quine [3], [4], Karnaugh [5], Nelson [6], [7] etc.. This paper presents an approach which is suitable for mechanical or automatic computation, as the Harvard method and the Quine method are so. On the other hand, it shares the same property as the Veitch method in the sense that some of essential prime implicants may be found before all prime implicants are computed. It also adopts the procedure to reduce the necessary steps for computation which is shown in Lawler [8]. The method described is applicable to the interval of Boolean functions f, g such that f implies g where for simplification of sum form the variables occurring in g also occur in f and for product form the variables in f also occur in g.

§ 1. Terminology and theorems

A logical variable with or without $-$ (negation) is called a primary. We make the conventions: f, g, and h with or without suffixes are Boolean functions which are neither identically equal to 1 nor identically equal to 0 and φ, x with or without suffixes are Boolean functions which have the just mentioned property and are constructed from primaries by a finite number of applications of $+$ (multi-variable or), $\cdot$ (multi-variable and) and (if necessary) the auxiliary symbols (,).

We write $\varphi_1, \varphi_2, \ldots, \varphi_n$ in place of $\varphi_1 \cdot \varphi_2 \cdot \cdots \cdot \varphi_n$. For an f which is primary, we define f to be both a σ-form and a π-form. For any f which is not primary, f is defined to be a σ-form or π-form if the last application of logical operation is $+$ or $\cdot$ respectively. We call $\varphi_1 + \cdots + \varphi_n$ the sum of

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