A BOUNDARY THEOREM FOR TSUJI FUNCTIONS

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To Professor K. NOSHIRO on his sixtieth birthday

1. Introduction. Let D denote the unit disc $|z|<1$, C the unit circle $|z|=1$ and C_r the circle $|z|=r$. Corresponding to any function $w(z)$ meromorphic in D we denote by $w^*(z)$ the spherical derivative

$$w^*(z)=\frac{|w'(z)|}{1+|w(z)|^2}.$$

We say that $w(z)$ is a Tsuji function if the spherical length of the curve $w(C_r)$ is a bounded function in $0<r<1$, in other words, provided

$$\sup_{r\to 1}\int_0^{2\pi} w^*(re^{i\theta})\,rd\theta<\infty. \tag{1}$$

A rectilinear segment S lying in D except for one endpoint $e^{i\theta}$ on C is called a segment of Julia for w, provided in each open triangle in D having one vertex at $e^{i\theta}$ and meeting S, the function w assumes all values on the Riemann sphere except possibly two. A point $e^{i\theta}$ is a Julia point for w provided each rectilinear segment lying in D except for one endpoint at $e^{i\theta}$ is a segment of Julia for w.

Corresponding to each θ and each α ($|\alpha|<\pi/2$), let $S(\theta,\alpha)$ be the segment that joins the points $e^{i\theta}$ and $(1-e^{i\alpha}\cos\alpha)e^{i\theta}$; in other words, let $S(\theta,\alpha)$ denote the chord of the circle with diameter $[0, e^{i\theta}]$ that forms a directed angle α with $[0, e^{i\theta}]$ at $e^{i\theta}$. In case $w(z)$ approaches a limit as $z\to e^{i\theta}$ on $S(\theta,\alpha)$, we denote this limit by $w(\theta,\alpha)$.

Further, let $L(\theta,\alpha)$ denote the spherical length of the image under the mapping w of $S(\theta,\alpha)$.

Tsuji [1] proved the following theorem.

For almost all θ, the radius $S(\theta, 0)$ is mapped on a rectifiable curve on the w-sphere so that a finite radial limit exists.

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