REGULAR TSUJI FUNCTIONS WITH INFINITELY MANY JULIA POINTS

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To K. NOSHIRO on his 60th birthday

1. Introduction

Let D denote the unit disk $|z|<1$, and C the unit circle $|z|=1$. Corresponding to any function f meromorphic in D we denote by f^* the spherical derivative

$$f^*(z)=\frac{|f'(z)|}{1+|f(z)|^2}.$$

We write

$$L(r)=\int_0^{2\pi} f^*(re^{i\theta})rd\theta, \qquad 0<r<1,$$

and shall say that $f\in T_1(l)$ if

$$\overline{\lim_{r\to 1}}\, L(r)\leq l<+\infty.$$

The functions $f\in T_1(l)$ are called Tsuji functions by Collingwood and Piranian [1]. Following their notation we call a rectilinear segment S lying in D except for one end-point $e^{i\theta}$ on C a segment of Julia for f provided that in each open triangle in D having one vertex at $e^{i\theta}$ and meeting S, the function f assumes all values on the Riemann sphere except possibly two. A point $e^{i\theta}$ is called a Julia point for f provided that each rectilinear segment S lying except for one endpoint $e^{i\theta}$ in D is a segment of Julia for f.

Following Tsuji [3] Collingwood and Piranian [1] investigated the class $T_1(l)$ and provided a number of illuminating examples. They proved among other results [1, Theorems 1, 5]

THEOREM A. *There exists a meromorphic Tsuji function for which each point of C is a Julia point.*

THEOREM B. *The function*

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