DEFINITIONS FOR A CLASS OF PLANE QUASICONFORMAL MAPPINGS

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Dedicated to Professor K. Noshiro on his sixtieth birthday

1. Introduction. This report is a survey of some of the many different ways of characterizing a class of plane quasiconformal mappings. This class was considered by Ahlfors [4] in his treatment of the Teichmüller problem, and it has been studied rather extensively in the last ten years.

In order to simplify notation, we shall consider only mappings between finite plane domains. Hence in what follows, D and D' will denote domains in the finite plane R^2, and f will denote an orientation preserving homeomorphism of D onto D'. Next for each quantity δ associated with D, such as a point, subset, or family of subsets, we shall let δ' denote its image under f. Finally given $1 \leq K < \infty$, we let Q_K denote the class of all K-quasiconformal homeomorphisms f, and Q the union of the classes Q_K for $1 \leq K < \infty$.

In the following sections we first list nine equivalent definitions for the class Q_K. Then we give three additional definitions for the class Q in the special case where $D = D' = R^2$. Finally we indicate where one may find proofs for the equivalence of these definitions.

2. Closure of the class of differentiable quasiconformal mappings. Given $1 \leq K < \infty$, we let Q_K^* denote the class of diffeomorphisms f which map each infinitesimal circle in D onto an infinitesimal ellipse with axis ratio bounded above by K. Then Q_K^* is the class of K-quasiconformal mappings studied by Grötzsch and Teichmüller, and it is well known that many important extremal problems for mappings in Q_K^* have solutions in Q_K^*. This fact is, at first, rather surprising since Q_K^* is obviously not closed under uniform convergence

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