ON THE ZEROS OF POWER SERIES WITH HADAMARD GAPS

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Dedicated to KIYOSHI NOSHIRO on his 60th birthday

1. Let

$$f(z)=c_0+\sum_{k=1}^{\infty}c_k z^{n_k} \tag{1}$$

be a power series with Hadamard gaps,

$$n_{k+1}/n_k \geq q > 1 \qquad (k \geq 1), \tag{2}$$

convergent in $|z|<1$.

In 1963 G. and M. Weiss [1] proved that $f(z)$ assumes every value infinitely often in $|z|<1$, if the constant q in (2) satisfies

$$q>q_0(\approx 100)$$

and

$$\sum |c_k| = \infty$$

In 1964 Ch. Pommerenke [2] showed that $f(z)$ assumes every value, at least once, if (2) holds for some $q>1$ and

$$\limsup_{k\to\infty}|c_k|>0. \tag{3}$$

The purpose of this paper is the proof of

THEOREM 1. *Let $f(z)$ be given by* (1), *let* (2) *be satisfied for some $q>1$ and suppose that* (3) *holds.*

Then $f(z)$ assumes every value infinitely often in $|z|<1$.

2. The proof is based on a lemma whose simplest form ($p=1$) was already used by G. and M. Weiss and on the idea, due to Hardy and Littlewood, of accentuating the dominance of the largest term of the series (1) by successive

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