ON q-TH DERIVATIVE OF VECTOR BUNDLES

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To Professor KIYOSHI NOSHIRO on the occasion of his 60th birthday

(0. 0) In the present note we shall be concerned with the improvement of fundamental definitions in higher order enumerative geometry which has been recently given by W. F. Pohl. Pohl's definition of q-th derivative of vector bundle is very complicated. We shall give a simpler and more reasonable definition of the q-th derivative of vector bundle in terms of sheaf theory and simplify the proofs in [P]. We shall also give a definition of higher order singularity of map.

1. The sheaf D_q

(1. 1) Let (X, O_X) be an object of a category C of ringed spaces over ground field k. Ringed space over k is a pair (X, O_X) of topological space X and sheaf O_X of k-algebras over X. (cf. [G]).

The following are most important examples of the category C;

(i) the category of differentiable-manifolds; k is the field R of real numbers.

(ii) the category of complex manifolds; k is the field $\mathbf{C}$ of complex numbers.

(iii) the category of algebraic k-schemes (or varieties defined over k).

(1. 2) Let F_1 be the sheaf of derivations of O_X over k, i.e.

$$F_1 = \{t \in \mathrm{Hom}_k(O_X, O_X) \mid t(ab) = a \cdot t(b) + bt(a) \text{ for any } a, b \in O_X \text{ and } t(c) = 0 \text{ for any } c \in k\}$$

where $\mathrm{Hom}_k(O_X, O_X)$ denotes the sheaf of k-homomorphisms of O_X into itself.

Then, F_1 is a sheaf of k-modules. Tensoring O_X with F_1 over k, we get a sheaf of O_X-modules $F = F_1 \otimes_k O_X$.

Although we may also consider F_1 as a sheaf of O_X-modules, we do not

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