THE THEOREM OF IDENTITY FOR COHERENT ANALYTIC MODULES

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To Professor KIYOSHI NOSHIRO on the occasion of his 60th birthday

1. Introduction. The theorem of identity for analytic subsets of a reduced complex space is stated as follows;

Let V and V' be two analytic subsets of a reduced complex space. If V is irreducible and there exists a point $x \in V$ such that the germ V_x of V at x is included in V'_x, then V is included in V'.

To an analytic set V there corresponds exactly one coherent Ideal $\mathscr{A}$ with rad $\mathscr{A} = \mathscr{A}$. Replacing the above statement concerning analytic sets by coherent Ideals, we obtain the theorem of identity for coherent Ideals with suitable conditions. The first result of this paper is to generalize this to the case of coherent Modules over a complex space $X = (|X|, \mathscr{O})$.

Let $\mathscr{L}$ be an arbitrarily given coherent $\mathscr{O}$-Module over X. We shall call a coherent sub-$\mathscr{O}$-Module $\mathscr{M}$ of $\mathscr{L}$ over X to be primary if $\mathscr{M}_x$ has no embedded primary component for any $x \in |\mathscr{L}/\mathscr{M}| := \{x\ ;\ (\mathscr{L}/\mathscr{M})_x \neq 0\}$ and the analytic subset $|\mathscr{L}/\mathscr{M}|$ is irreducible.

We shall prove in § 3

THEOREM (1st theorem of identity). *If $\mathscr{M}$ is primary and $\mathscr{M}_x \supseteq \mathscr{N}_x$ for another coherent sub-$\mathscr{O}$-Module $\mathscr{N}$ of $\mathscr{L}$ and some $x \in |\mathscr{L}/\mathscr{M}|$, we have $\mathscr{M} \supseteq \mathscr{N}$.*

For an arbitrary $\mathscr{M}$, we shall show

THEOREM (2nd theorem of identity). *For an arbitrarily given coherent sub-$\mathscr{O}$-Module $\mathscr{M}$ of $\mathscr{L}$ there exists a locally finite family of irreducible analytic sets $\{V_\iota\}$ such that any coherent sub-$\mathscr{O}$-Module $\mathscr{N}$ of $\mathscr{L}$ with $\{x\ ;\ \mathscr{M}_x \supseteq \mathscr{N}_x\} \cap V_\iota \neq \phi$ for any ι is contained in $\mathscr{M}$* (§ 5).

To seek such $\{V_\iota\}$ we pay attention to a reduced primary decomposition of $\mathscr{M}_x$ in $\mathscr{L}_x$ for $x \in |\mathscr{L}/\mathscr{M}|$. The class of all prime ideals associated with

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