GENERALIZATION OF LEVI-OKA'S THEOREM CONCERNING MEROMORPHIC FUNCTIONS

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Dedicated to Professor K. NOSHIRO on his sixtieth birthday

Introduction

As Fuks [3] stated, every domain of holomorphy or meromorphy over C^n is analytically convex in the sense of Hartogs. Oka [6] proved that every domain over C^n analytically convex in the sense of Hartogs is a domain of holomorphy. Therefore a domain of meromorphy over C^n coincides with a domain of holomorphy over C^n.

In the present paper we shall prove that the envelope of meromorphy of a domain (D, φ) over a Stein manifold S with respect to a family of meromorphic functions on D is p_7-convex in the sense of Docquier-Grauert [2] and, therefore, is a Stein manifold. Especially a domain of meromorphy over S coincides with a domain of holomorphy over S.

A complex manifold M is called of *weak* (or *strong*) *Poincaré type* if for any meromorphic function f on M there exist holomorphic functions g and h on M such that $f=g/h$ on M (and that g and h are coprime at each point of M). From Siegel [8] any complex manifold of Cousin-II type is of strong Poincaré type and from Hitotumatu-Kôta [4] any Stein manifold is of weak Poincaré type.

Let (D, φ) be a domain over a Stein manifold and f be a meromorphic function on D. There exists a meromorphic function $\tilde{f}$ on the domain $(\tilde{\lambda}_f, \tilde{D}_f, \tilde{\varphi}_f)$ of meromorphy of f such that $f=\tilde{f}\circ\tilde{\lambda}_f$. As $\tilde{D}_f$ is a Stein manifold which is of weak Poincaré type, there exist holomorphic functions $\tilde{g}$ and $\tilde{h}$ on $\tilde{D}_f$ such that $\tilde{f}=\tilde{g}/\tilde{h}$ on $\tilde{D}_f$. Then holomorphic functions $g=\tilde{g}\circ\tilde{\lambda}_f$ and $h=\tilde{h}\circ\tilde{\lambda}_f$ on D satisfies $f=g/h$ on D. This means that any domain over a Stein manifold is of weak Poincaré type.

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