NOTE ON THE UNIQUENESS PROPRETY OF WEAK SOLUTIONS OF PARABOLIC EQUATIONS

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To Professor KIYOSHI NOSHIRO on the occasion of his 60th birthday

1. Aronson proved, in his paper [1], the existence and the uniqueness property of weak solutions of the initial boundary value problem for parabolic equations of second order with measurable coefficients. On the uniqueness of solutions of the Cauchy problem for such equations he also gave some interesting results in [2].

In this note we prove the uniqueness property of weak solutions of the initial boundary value problem for some equations of higher order by applying the argument used in [2].

2. We denote by x a point $(x_1, \ldots, x_n)$ in the n-dimensional Euclidean space R^n and by t a point on the real line $(-\infty, \infty)$. Let $\mathscr{D}$ be a bounded domain given in R^n and let $\overline{\mathscr{D}}$ be its closure. We denote by Ω the cylinder domain $\mathscr{D}\times(T', T'')$ in the $(n+1)$-dimensional Euclidean space $R^n\times(-\infty, \infty)$.

We introduce some function spaces.

The space $H_0^{s,2}(\mathscr{D})$ is the closure of $C_0^\infty(\mathscr{D})$ by the norm

$$\|\varphi\|_s=\Big(\int_{\mathscr{D}}\sum_{|\alpha|\leqq s}|D_x^\alpha\varphi|^2dx\Big)^{1/2}.$$

The space $L^2[T', T''; H_0^{s,2}(\mathscr{D})]$ consists of all functions u with the following properties: i) u is measurable in Ω, ii) for almost all $t\in[T', T'']$, the function $u(x, t)$ in x belongs to $H_0^{s,2}(\mathscr{D})$ and iii) the norm $\|u\|_s$ as a function of t belongs to $L^2([T', T''])$.

We have the definition of the space $H^{1,2}[T', T''; H_0^{s,2}(\mathscr{D})]$ if, in the above definition of $L^2[T', T''; H_0^{s,2}(\mathscr{D})]$, the condition iii) is replaced by iii)′: the norm $\|u\|_s$ as a function of t belongs to $H^{1,2}([T', T''])$, which is the closure of $C^\infty((T', T''))$ by the norm

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