ON THE CLASS NUMBER OF A RELATIVELY CYCLIC NUMBER FIELD

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To Professor KIYOSHI NOSHIRO on the occasion of his 60th birthday

Introduction

Let l be a rational prime. For each $n \geqq 0$, denote by ζ_{l^n} a primitive l^n-th root of unity and by $\mathbf{Q}(\zeta_{l^n})$ the cyclotomic field obtained by adjoining ζ_{l^n} to the rational field $\mathbf{Q}$. Then a theorem which was proved by H. Weber[1] is well known:

THEOREM (H. WEBER). *The class number of* $\mathbf{Q}(\zeta_{2^\nu})$ *is odd.*

As a generalization of this theorem of Weber, Ph. Furtwängler[2] gave:

THEOREM (PH. FURTWÄNGLER). *The class number of* $\mathbf{Q}(\zeta_{l^\nu})$ *is divisible by the prime* l *if and only if the class number of* $\mathbf{Q}(\zeta_l)$ *is divisible by* l.

Moreover, Ph. Furtwängler[3] obtained

THEOREM (PH. FURTWÄNGLER). *Let* F *and* K *be two subfields of* $\mathbf{Q}(\zeta_{l^\nu})$. *If* F *is contained in* K, *then the class number of* K *is divisible by the class number of* F.

Afterwards, K. Iwasawa[4] generalized these theorems, and got

THEOREM (K. IWASAWA)[5]. *Let* F *be an algebraic number field, and let* K *be a finite Galois extension of* F. *Then we have the following facts:*

(I) If there exists a prime divisor P *of* F *which is fully ramified in the extension* K/F, *then the class number of* K *is divisible by the class number of* F.

(II) If, furthermore, K/F *is a cyclic extension of prime power degree* l^ν *and*

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1) Cf. H. Weber [21].

2) Cf. Ph. Furtwängler [7].

3) Cf. Ph. Furtwängler [6].

4) Cf. K. Iwasawa [12].

5) This theorem is often referred to e.g. in S.-N.Kuroda [16], K. Iwasawa [14] etc.