ON A THEOREM OF SCHWARZ TYPE FOR QUASICONFORMAL MAPPINGS IN SPACE

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To Professor KIYOSHI NOSHIRO on the occasion of his 60th birthday

A space ring R is defined as a domain whose complement in the Moebius space consists of two components. The modulus of R can be defined in variously different but essentially equivalent ways (see e.g. Gehring [3] and Krivov [5]), which is denoted by mod R. Following Gehring [2], we refer to a homeomorphism $y(x)$ of a space domain D as a K-quasiconformal mapping, if the modulus condition

$$(*)\qquad \frac{1}{K} \bmod R \leqq \bmod y(R) \leqq K \bmod R$$

is satisfied for all bounded rings R with their closure $\overline{R} \subset D$, where $y(R)$ denotes the image of R by $y = y(x)$. Then, it is evident that the inverse of a K-quasiconformal mapping is itself K-quasiconformal and that a K_1-quasiconformal mapping followed by a K_2-quasiconformal one is $K_1 K_2$-quasiconformal. It is also well known that the restriction of a Moebius transformation to a space domain is equivalent to a 1-quasiconformal mapping of its domain.

The purpose of this paper is to prove Theorem 2 in the previous paper [4] (see also corrections to it added after the list of references in this paper) without the additional condition "$y(x)$ maps each radius of $|x| < 1$ onto a curve which is normal to the image of each surface $|x| = r$" and without the use of any isoperimetric inequality such as $A(r)^3 - 36\pi V(r)^2 \geqq 0$ used in its former proof, and to give the various space forms derived from there. All our arguments can be similarly carried over to higher dimensions, but we shall restrict ourselves for brebity sake to the Moebius 3-dimensional space.

1. First we enunciate the theorem.

THEOREM. *Let $y = y(x)$ be a K-quasiconformal mapping of $|x| < 1$ such that*

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