SOME RESULTS AND PROBLEMS CONCERNING CHORDAL PRINCIPAL CLUSTER SETS*

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To Professor KIYOSHI NOSHIRO on the occasion of his 60th birthday

Let Γ be the unit circle and D be the open unit disk in the complex plane, and denote the Riemann sphere by Ω. By an arc at a point $\zeta \in \Gamma$ we mean a continuous curve Λ: $z = z(t)$ $(0 \leqq t < 1)$ such that $|z(t)| < 1$ for $0 \leqq t < 1$ and $\lim_{t \to 1} z(t) = \zeta$. A terminal subarc of an arc Λ at ζ is a subarc of the form $z = z(t)$ $(t_0 \leqq t < 1)$, where $0 \leqq t_0 < 1$. Suppose that $f(z)$ is a meromorphic function in D. Then $A(f)$ denotes the set of asymptotic values of f; and if $\zeta \in \Gamma$, then $C(f, \zeta)$ means the cluster set of f at ζ and $C_{\mathscr{A}}(f, \zeta)$ is the outer angular cluster set of f at ζ (see [13]). The principal cluster set of f at ζ is the set

$$\Pi(f, \zeta) = \bigcap_{\Lambda} C_{\Lambda}(f, \zeta),$$

where Λ ranges over all arcs at ζ. As is well known, this set is of importance, and was introduced some time ago, in connection with the theory of boundary correspondence under conformal mapping. More recently the set

$$\Pi_X(f, \zeta) = \bigcap_{X} C_X(f, \zeta),$$

where X ranges over all chords of the unit circle at ζ, has received attention, notably in the work of Meier [12], who has used this set, which we call the chordal principal cluster set of f at ζ, in the formulation of his topological analogue of Plessner's theorem.

Because of the significance of the chordal principal cluster set in this connection as well as others, the present paper is devoted to a more systematic investigation of this set for its own sake as well as its relation to the principal cluster set.

It is evident to begin with that both $\Pi(f, \zeta)$ and $\Pi_X(f, \zeta)$ are closed subsets of Ω, and that

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