EQUICONTINUITY ON HARMONIC SPACES

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Dedicated to Professor Kiyoshi Noshiro on his 60th birthday

G. Mokobodzki proved [5] that on any harmonic space with countable basis satisfying the axioms 1, 2, T_+, K_D [2] [1] any equally bounded set of harmonic functions is equicontinuous. P. Loeb and B. Walsh showed [4] that the same property holds on a harmonic space without countable basis, if Brelot's axiom 3 is fulfilled. The aim of this paper is to generalize these results to a harmonic space X satisfying only the axioms 1, 2_0, K_1, [2] [1] where 2_0 is a weakened form of axiom 2. As a corollary we get: if any point of X possesses two open neighbourhoods U, V such that the set of harmonic functions on U separates the points of $U \cap V$, then X has locally a countable basis.

Throughout this paper Bourbaki's notations and terminology will be used.

1. Family of measures

Throughout this paragraph we shall denote by X, Y two compact spaces and by $(\omega_x)_{x \in X}$ a family of (nonnegative) measures on Y such that for any equally bounded upper directed family $(f_\iota)_{\iota \in I}$ of Borel functions on Y the function on X

$$x \to \sup_{\iota \in I} \int f_\iota d\omega_x$$

is continuous. We denote for any bounded Borel function f on Y by f' the function on X

$$x \to \int f d\omega_x.$$

It is a continuous function. We denote further for any measure μ on X by μ' the measure on Y

$$f \to \int f' d\mu \qquad (f \in \mathscr{K}(Y)).$$

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