THE SCHUR MULTIPLIERS OF THE MATHIEU GROUPS[1]

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To the memory of TADASI NAKAYAMA

The Mathieu groups are the finite simple groups M_{11}, M_{12}, M_{22}, M_{23}, M_{24} given originally as permutation groups on respectively 11, 12, 22, 23, 24 symbols. Their definition can best be found in the work of Witt [1]. Using a concept from Lie group theory we can describe the Schur multiplier of a group as the center of a "simply-connected" covering of that group. A precise definition will be given later. We also mention that the Schur multiplier of a group is the second cohomology group of that group acting trivially on the complex roots of unity. The purpose of this paper is to determine the Schur multipliers of the five Mathieu groups. We conclude:

the multipliers of M_{11}, M_{12}, M_{23}, M_{24} are trivial;
the multiplier of M_{22} is a cyclic group of order 3.

The proof of this result occupies part II of this paper. The techniques used there differ somewhat from those introduced by I. Schur in his three impressive memoirs [2]. We hope that our methods may have an independent interest.[*]

In part I we collect certain elementary (and undoubtedly well-known) results on the Mathieu groups. Several of these results are necessary for the proofs in part II.

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PART I

Frobenius, in § 5 and § 6 of his work [3] on multiply-transitive groups,

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[*] Added in proof. These results have also been obtained by J. G. Thompson.