GALOIS THEORY FOR RINGS WITH FINITELY MANY IDEMPOTENTS

O. E. VILLAMAYOR and D. ZELINSKY[1]

To the respected memory of TADASHI NAKAYAMA

0. Introduction

In [5], Chase, Harrison and Rosenberg proved the Fundamental Theorem of Galois Theory for commutative ring extensions $S \supset R$ under two hypotheses: (i) S (and hence R) has no idempotents except 0 and 1; and (ii) S is Galois over R with respect to a finite group G—which in the presence of (i) is equivalent to (ii′) : S is separable as an R-algebra, finitely generated and projective as an R-module, and the fixed ring under the group of all R-algebra automorphisms of S is exactly R. We shall refer to the Fundamental Theorem under these hypotheses as "CHR Galois Theory." This terminology is not quite just to Chase, Harrison and Rosenberg, since even if S has idempotents, they have a Fundamental Theorem, but hypothesis (ii) now requires that a finite group G be given (definitely not the group of all automorphisms of S) having R as fixed ring, and satisfying the Galois hypothesis of [2, p. 396]. Furthermore, this Fundamental Theorem gives a one-to-one correspondence between subgroups of this given group and *some* separable subalgebras of S.

In this note, we propose an alternative approach when R (or, rather, the image of R in S) has finitely many idempotents, and when S and R satisfy hypothesis (ii′). These are hypotheses only on S and R and not on a prescribed group of automorphisms (It is true that in this case S is Galois over R with respect to a finite group, in fact, with respect to several different finite groups. It is partly this multiplicity of groups that prompted our investigation). Our conclusions give a one-to-one correspondence between all projective, separable subalgebras of S and some subgroups (called "fat" subgroups) of the full automorphism group of S over R. The fat subgroups are easily describable in

Received August 26, 1965.

[1] This work was supported by NSF grants GP 1649 and GP 3895.