THE COHOMOLOGY GROUPS OF TORI IN FINITE GALOIS EXTENSIONS OF NUMBER FIELDS

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To the memory of TADASI NAKAYAMA

Class field theory determines in a well-known way the higher dimensional cohomology groups of the idèles and idèle classes in finite Galois extensions of number fields. At the Amsterdam Congress in 1954 I announced [7] the corresponding result for the multiplicative group of the number field itself, but the proof has never been published. Meanwhile, Nakayama showed that results of this type have much broader implications than had been realized. In particular, his theorem allows us to generalize our result from the multiplicative group to the case of an arbitrary torus which is split by the given Galois extension. We also treat the case of "S-units" of the multiplicative group or torus, for a suitably large set of places S. It is a pleasure for me to publish this paper here, in recognition of Nakayama's important contributions to our knowledge of the cohomological aspects of class field theory; his work both foreshadowed and generalized the theorem under discussion.

Notations, and the plan of the proof. Let L be an algebraic number field of finite degree, or an algebraic function field of one variable over a finite constant field, and let K/L be a finite Galois extension with group G. By a *place* of K we mean an equivalence class of non-trivial absolute values, archimedean or non-archimedean. Let S be a set of places of K satisfying the following conditions:

($S1$) S is stable under G.

($S2$) S contains all archimedean places.

($S3$) S contains all places ramified over L.

($S4$) S is large enough so that every ideal class of K contains an ideal with support in S.

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