COMPLETELY FAITHFUL MODULES AND SELF-INJECTIVE RINGS

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Dedicated to the memory of Professor TADASI NAKAYAMA

A left module over a ring Λ is called completely faithful if Λ is a sum of those left ideals which are homomorphic images of M. The notion was first introduced by Morita [9][1], and he proved, among others, the following theorem which plays a basic role in his theory of category-isomorphisms: if a Λ-module M is completely faithful, then M is finitely generated and projective with respect to the endomorphism ring Γ of M and Λ coincides with the endomorphism ring of Γ-module M. § 1 of the present paper is devoted to summerize, with some supplements and refinements, Morita's theorems centering around the notion of completely faithful modules. Now, the above theorem implies in particular every completely faithful modules is faithful. However the converse is not necessarily true, so that there naturally arises a problem to find out possible types of ring Λ for which every faithful module is completely faithful. In § 2 we shall give a complete answer to the problem: in order that every faithful left module be completely faithful it is necessary and sufficient that Λ be left self-injective and a direct sum of indecomposable left ideals having minimal left subideals. The rings characterized here may be regarded as a natural extension of quasi-Frobenius rings which have been largely studied by Nakayama [11, 12].[2] In fact, the theorem of Nesbitt and Thrall [14] that every faithful representation of a quasi-Frobenius algebra contain the reduced regular representation as a direct constituent means actually that quasi-Frobenius algebras provide a typical example for the problem.

1. Let Λ be a ring with identity element 1. Let $M = {}_{\Lambda}M$ be a (unital) left Λ-module. If we let f range over all Λ-homomorphisms of M into Λ, the

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1) Cf. also Appendix of Auslander-Goldman [1].

2) Cf. also Eilenberg-Nakayama [6] and Ikeda [7].