ORTHOGONAL GROUP MATRICES OF HYPEROCTAHEDRAL GROUPS

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To the memory of TADASI NAKAYAMA

1. Introduction. The hyperoctahedral group G_n of order $2^n n!$ is generated by permutations and sign changes applied to n digits, $d=1, 2, \ldots, n$. The 2^n sign changes generate a normal subgroup Σ_n whose factor group G_n/Σ_n is isomorphic with the symmetric group S_n of order $n!$. To each irreducible orthogonal representation $\{\lambda ; \mu\}$ of G_n corresponds an ordered pair of partitions $[\lambda]$ of l and $[\mu]$ of m, where $l+m=n$. The faithful representation $\{n-1 ; 1\}$ of G_n is the real monomial group R_n of degree n. The representations $\{\lambda ; 0\}$ of G_n with $l=n$, $m=0$, are isomorphic with corresponding irreducible representations $\{\lambda\}$ of S_n. If the representation $\{\lambda ; \mu\}$ maps the element g_k of G_n into the real orthogonal matrix $M^{\lambda\mu}(g_k)$ of degree $f^{\lambda\mu}$, we define the *group matrix* of $\{\lambda ; \mu\}$ to be

$$\mathfrak{M}^{\lambda\mu} = \sum_k g_k^{-1} M^{\lambda\mu}(g_k) \qquad g_k \in G_n \tag{1.1}$$

Our purpose is to determine explicitly for each $\{\lambda ; \mu\}$ the uv-entry of the group matrix of an irreducible orthogonal representation of G_n, and incidentally those of S_n, in the form

$$\mathfrak{M}^{\lambda\mu}_{uv} = \gamma_v E^{\lambda\mu} \sigma^{\lambda\mu} \gamma_u^{-1} \tag{1.2}$$

by describing in the group ring Γ of G_n a suitable pair of ring elements $E^{\lambda\mu}$ related to permutations of S_n, and $\sigma^{\lambda\mu}$ related to sign changes of Σ_n, and also a set of invertible ring factors γ_v that meet our requirements. Matrices $M^{\lambda 0}(\tau_d)$ for transpositions τ_d of consecutive digits d, $d+1$ are to be those of Young's orthogonal representation $\{\lambda\}$ of S_n [4]. The matrix $M^{\lambda\mu}(\sigma_d)$ for the element σ_d of Σ_n that changes the sign of the digit d is to be a diagonal matrix with vv-entry $+1$ or -1 according as the digit d is assigned to the

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