A THEOREM ON ANALYTIC MAPPINGS OF COMPLEX MANIFOLDS

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Dedicated to late professor TADASI NAKAYAMA

We prove in this paper a theorem on analytic mappings of the complex space C_n into the complex projective space P_n. The theorem is closely related to that of S. S. Chern in [1], and the main idea of the proof is the same with the latter, though the calculations are rather different. The background of our calculation is the normal contact metric structure which was found by S. Sasaki and Y. Hatakeyama [4].

Our purpose is to find a criterion for an analytic mapping f of C_n into P_n in order that $f(C_n)$ covers almost every part of P_n. We take cartesian coordinates $z^1, \ldots, z^n$ in C_n and then the metric is given by

$$d\Sigma^2 = \sum_{j=1}^{n} dz^j d\bar{z}^j. \tag{0.1}$$

As for the elliptic metric of P_n we have

$$dT^2 = (1 + |w|^2)^{-2} \Big(\sum_{j=1}^{n} dw^j d\overline{w}^j + \sum_{j<k} | dw^j w^k - dw^k w^j |^2 \Big) \tag{0.2}$$

in complex coordinates $w^1, \ldots, w^n$, where we have put $|w| = (\sum w^j \overline{w}^j)^{1/2}$. An analytic mapping $f : C_n \to P_n$ can be represented by

$$w^j = f_j(z^1, \ldots, z^n) \qquad (j = 1, \ldots, n), \tag{0.3}$$

where $f_j(z^1, \ldots, z^n)$ are analytic functions. We put

$$f^*(dT^2) = \sum_{jk} a_{jk} dz^j d\bar{z}^k. \tag{0.4}$$

(a_{jk}) is a hermitian tensor on C_n which is determined by the mapping f. We denote the eigenvalues of (a_{jk}) by $\lambda_1, \ldots, \lambda_n$ and put

$$B = \sum_{j=1}^{n} \lambda_1 \lambda_2 \cdots \lambda_{j-1} \lambda_{j+1} \cdots \lambda_n. \tag{0.5}$$

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