HOLOMORPHIC COHOMOLOGY OF COMPLEX ANALYTIC LINEAR GROUPS

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To the memory of TADASI NAKAYAMA

Introduction. Let G be a complex analytic group, and let A be the representation space of a finite-dimensional complex analytic representation of G. We consider the cohomology for G in A, such as would be obtained in the usual way from the complex of holomorphic cochains for G in A. Actually, we shall use a more conceptual categorical definition, which is equivalent to the explicit one by cochains. In the context of finite-dimensional representation theory, nothing substantial is lost by assuming that G is a linear group. Under this assumption, it is the main purpose of this paper to relate the holomorphic cohomology of G to Lie algebra cohomology, and to the rational cohomology, in the sense of [1], of algebraic hulls of G. This is accomplished by using the known structure theory for complex analytic linear groups in combination with certain easily established results concerning the cohomology of semidirect products. The main results are Theorem 4.1 (whose hypothesis is always satisfied by a complex analytic linear group) and Theorems 5.1 and 5.2. These last two theorems show that the usual abundantly used connections between complex analytic representations of complex analytic groups and rational representations of algebraic groups extend fully to the superstructure of cohomology.

1. Holomorphic maps. We recall some of the elementary facts concerning (Hausdorff) topological vector spaces over the field C of the complex numbers. Every finite-dimensional subspace of such a topological vector space is closed and topologically isomorphic with a C^n. If A is a locally convex topological vector space over C then every neighborhood of 0 in A contains a neighborhood of 0 that is not only convex but also stable under the multiplications with complex numbers of absolute value not exceeding 1. Such a neighborhood is

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