ON SOME DUALITIES CONCERNING ABELIAN VARIETIES

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Dedicated to late Professor Tadasi Nakayama

Introduction. The group of extensions $\mathrm{Ext}(A, G_a)$ and $\mathrm{Ext}(A, G_m)$ of an abelian variety A by the additive or multiplicative group G_a, G_m have been investigated in detail ([9], [10], [GACC]). On the other hand, F. Oort [8] and M. Miyanishi [6] recently studied the groups $\mathrm{Ext}(G_a, A)$ and $\mathrm{Ext}(G_m, A)$. The purpose of the present paper is to clarify the relationship between these groups. Our results show that the latter groups can be derived from the former.

Sections 1 and 2 are preliminaries. Section 3 contains our main result, namely the existence of a canonical duality between the vector spaces $\mathrm{Ext}(A, G_a)$ and $\mathrm{Ext}(G_a, A)$. The functorial property of the pairing of duality is also proven. In Section 4 the functoriality is used to prove that the pairing behaves nicely with respect to the Frobenius operator. A proof of a theorem concerning $H^1(A, \mathcal{O})$ is added. In the last Section 5 a similar duality is defined for $\mathrm{Ext}(A, G_m)$ and $\mathrm{Ext}(G_m, A)$; this is much easier. As the consequence we see that $\mathrm{Ext}(G_m, A)$ is the character group of the Tate group $T(A^*)$ (cf. [5] Ch. VII) of the dual abelian variety A^*.

Though our results are in the negative direction (no new functors!), they seem to have some interest in view of the various duality phenomena in the theory of commutative algebraic groups, for which unified treatments are being made by several mathematicians (cf. [1], [2]).

We shall use the definitions and the results about group extensions contained in Serre's book [GACC].

1. The following results are known:

(i) $\mathrm{Ext}(A, G_m)$ is canonically isomorphic to the underlying group of the dual abelian variety (i.e. the Picard variety) A^* of A, while $\mathrm{Ext}(G_m, A)$ is

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