

CLIFFORD ALGEBRAS AND FAMILIES OF ABELIAN VARIETIES

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In the arithmetic theory of automorphic functions on a symmetric bounded domain $\mathscr{D} = G/K$, as developed recently by Shimura and Kuga [2], [2a], it is important to consider a family of (polarized) abelian varieties on \mathscr{D} obtained from a symplectic representation ρ (defined over \mathbf{Q}) of G (viewed as an algebraic group defined over \mathbf{Q}) satisfying a certain analyticity condition. Recently, I have determined completely such representations, reducing the problem to the case where G is a \mathbf{Q} -simple group and where ρ is a \mathbf{Q} -primary representation ([3], [4]). It has turned out that, besides the four standard solutions investigated already by Shimura, there exist two more non-standard solutions, one of which comes from a spin representation of the orthogonal group and thus gives a family of abelian varieties on a domain of type (IV). The purpose of this short note is to explain how one can construct most simply, starting from the "regular representation" of the corresponding Clifford algebra, examples of such families, including also the non-analytic case.

1. Let V be an n -dimensional vector-space over \mathbf{R} , provided with a non-degenerate symmetric bilinear form S of signature (p, q) . We denote by $C = C(V, S)$ the corresponding Clifford algebra, by C^+ (resp. C^-) its even (resp. odd) part, and define the "spin group" (or "reduced Clifford group" in the terminology of [1]) as follows:

$$(1) \quad G = \{g \in C^+ \mid g'g = 1, gVg^{-1} = V\},$$

$'$ denoting the canonical involution of C . We assume that $p, q > 0$, $n = p + q > 2$. The following Proposition is well-known (see e.g. [1], 2.9).

PROPOSITION 1. 1) *G is a connected semi-simple Lie group and the mapping*

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