INVOLUTIVE PROPERTY OF RESOLUTIONS OF DIFFERENTIAL OPERATORS

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Dedicated to the Memory of Professor TADASI NAKAYAMA

§ 0. Introduction

Let E and E' be C^∞ vector bundles over a C^∞ manifold M. Denote by $\Gamma(E)$ (resp. by $\Gamma(E')$) the vector space of C^∞ cross-sections of E (resp. of E') over M. Take a linear differential operator of the first order $D : \Gamma(E) \to \Gamma(E')$ induced by a vector bundle mapping $\sigma(D) : J^1(E) \to E'$, where $J^k(E)$ denotes the vector bundle of k-jets of cross-sections of E. Take an integer $l \geq 0$. Then $\sigma(D)$ induces a vector bundle mapping $\sigma^l(D) : J^l(J^1(E)) \to J^l(E')$. Now $J^{l+1}(E)$ can be canonically considered as a vector sub-bundle of $J^l(J^1(E))$. Denote by $B_{(l)}$ the image of $J^{l+1}(E)$ by $\sigma^l(D)$. In the case $B_{(l)}$ is a sub-bundle of $J^l(E')$ denote by $E''_{(l)}$ the quotient vector bundle $J^l(E')/B_{(l)}$. Then the canonical projection $J^l(E') \to E''_{(l)}$ induces a linear differential operator $D'_{(l)} : \Gamma(E') \to \Gamma(E''_{(l)})$. We set $E'' = E''_{(1)}$ and $D' = D'_{(1)}$ when they are defined.

We say that a differential operator D is involutive when the equations $Du = 0$ $(u \in \Gamma(E))$ is involutive. It is shown in [5] that if D is involutive then $B_{(l)}$ is a sub-bundle and $D'_{(l)}$ is involutive for sufficiently large l. The purpose of the present note is to show that B is a sub-bundle and D' is involutive.

The reason why such problem is considered is the following: If we consider in the category of real analyticity in stead of in the category of infinitely differentiability and if we assume that $B_{(l)}$ is a sub-bundle and that $D'_{(l)}$ is involutive, then the sequence

$$\Gamma_\omega(E) \to \Gamma_\omega(E') \to \Gamma_\omega(E''_{(l)})$$

is exact, where Γ_ω indicates the sheaf of germs of real analytic cross-sections and the first (resp. the second) arrow denotes D (resp. $D'_{(l)}$) (cf. [5]). Thus our result shows that a linear involutive differential operator D of the first order

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