

GENERATORS AND RELATIONS FOR CYCLOTOMIC UNITS

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To the memory of TADASI NAKAYAMA

1. Introduction

We prove here an unpublished conjecture of Milnor which gives a complete set of multiplicative relations between the numbers

$$e'(\zeta) = 1 - \zeta,$$

where $\zeta \neq 1$ ranges over complex roots of unity. Information of this type is useful in certain areas of topology as well as in number theory.

2. Statement of the theorem

Clearly

$$(A) \quad e'(\zeta^{-1}) = -\zeta^{-1}e'(\zeta).$$

Suppose $\zeta^n \neq 1$. In

$$t^n - 1 = \prod_{\eta^n=1} (t - \eta)$$

substitute ζ^{-1} for t to obtain

$$\zeta^{-n} - 1 = \prod_{\eta^n=1} \zeta^{-1}(1 - \zeta\eta),$$

and then multiply by ζ^n , yielding

$$(B) \quad e'(\zeta^n) = \prod_{\eta^n=1} e'(\zeta\eta) \quad \text{if } \zeta^n \neq 1.$$

MILNOR'S CONJECTURE. *All multiplicative relations, modulo torsion, between the $e'(\zeta)$, are consequences of (A) and (B) above.*

The following theorem is slightly more precise.

THEOREM 1. *Let U_m^1 denote the multiplicative group generated by all $e'(\zeta)$*

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