# REMARKS ON A THEOREM OF BOURBAKI

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In memory of TADASI NAKAYAMA

Bourbaki has established the following theorem which we state without proof as

THEOREM A ([3, Theorem 6, §4]). *Let $R$ be a noetherian integrally closed domain and $M$ a finitely generated torsion free $R$-module. Then there exists a free submodule $F$ of $M$ such that $M/F$ is isomorphic to an ideal in $R$.*

It is our purpose in this note to present a few consequences of this theorem. Before giving these results we briefly review some terminology and known results.

We shall assume throughout this paper that all rings are noetherian. Suppose $R$ is a local ring and $M$ is a finitely generated, non-zero $R$-module. Then a sequence of elements $x_1, \ldots, x_s$ in the maximal ideal $\mathfrak{m}$ of $R$ is called an *$M$-sequence* if $x_i$ is not a zero-divisor in $M/(x_1, \ldots, x_{i-1})M$ for $i = 1, \ldots, s$. It is easily seen that if $x_1, \ldots, x_s$ is an $M$-sequence, then $s \leq$ Krull dim $R$. Thus all $M$-sequences can be extended to maximal $M$-sequences. It is well known that all maximal $M$-sequences have the same length and that this length is the same as the smallest integer $i \geq 0$ such that $\mathrm{Ext}_R^i(R/\mathfrak{m}, M) \neq 0$ (see [2, Proposition 2.9] for instance). We shall denote by $\mathrm{codh}_R M$ the length of a maximal $M$-sequence.

We now list without proof some of the well-known basic properties of $\mathrm{codh}_R M$. These can easily be derived from the characterization of $\mathrm{codh}_R M$ in terms of the functor $\mathrm{Ext}_R^*(R/\mathfrak{m},)$.

LEMMA 1. *Let $R$ be a local ring with maximal ideal $\mathfrak{m}$ of dimension $d$. Let $0 \to M' \to M \to M'' \to 0$ be an exact sequence of non-zero finitely generated $R$-modules. Then we have*

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