A REMARK ON DIFFERENTIABLE STRUCTURES ON REAL PROJECTIVE $(2n-1)$-SPACES

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Dedicated to the memory of Professor TADASI NAKAYAMA

The main objective of this paper is to study the action of the group of differentiable structures Γ_{2n-1} on the $(2n-1)$-sphere S^{2n-1} on the diffeomorphism classes on the real projective $(2n-1)$-space P^{2n-1} by connected sum. This is done by considering universal covering spaces of the connected sum $P^{2n-1}\#\sum$, where $\sum$ is an exotic $(2n-1)$-sphere.

Throughout this paper all the manifolds considered are oriented, compact, and connected. Also the word *differetiable* is meant C^∞*-differentiable.*

1. Let M, N be n-dimensional differentiable manifolds. If there exists an orientation preserving diffeomorphism of M onto N, then we shall denote it by $M\approx N$. The manifold M with orientation reversed is denoted by $-M$.

Let $M_1\# M_2$ be the connected sum of two n-manifolds M_1 and M_2. It is known that the connected sum operation is associative and commutative up to orientation preserving diffeomorphism. The sphere S^n serves as identity element (Cf. J. Milnor [2], M. Kervaire- J. Milnor [1]).

Let $\sum$ be a smooth combinatorial n-sphere, which is called an exotic sphere. Then $\sum\#(-\sum)\approx S^n$. Thus the set of all the orientation preserving diffeomorphism classes of the exotic spheres forms a group under connected sum, which is denoted by Γ_n. We shall denote the class of $\sum$ by $\{\sum\}$.

Let M be a differentiable n-manifold. Let $\sum$ be an exotic n-sphere such that $M\#\sum\approx M$. Let $\Delta(M)$ be the subset of Γ_n consisting of the classes of such $\sum$.

PROPOSITION 1. *$\Delta(M)$ is a subgroup of Γ_n, and $M\#\sum_1\approx M\#\sum_2$, for exotic spheres $\sum_1$, $\sum_2$, if and only if $\{\sum_1\}-\{\sum_2\}\in\Delta(M)$.*

Proof. Let $\{\sum_1\}$, $\{\sum_2\}\in\Delta(M)$. Then

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