GLOBAL NORM-RESIDUE MAP OVER QUASI-FINITE FIELD

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Dedicated to the memory of Professor TADASI NAKAYAMA

A field k is called quasi-finite if it is perfect and if $G_k \approx \hat{Z}$ where G_k is the Galois group of the algebraic closure k_c over k and $\hat{Z}$ is the completion of the additive group of the rational integers. The classical reciprocity law on the local field with finite residue field is well-known to hold on local fields with quasi-finite residue field ([4], [5]). Thus it is natural to ask if the global reciprocity law should hold in the ordinary sense (see § 1 below) on the function-fields of one variable over quasi-finite field. We consider here two basic prototypes of non-finite quasi-finite fields:

(a) field k of non-zero characteristic which is algebraic over the prime subfield k_0 and has a finite p-primary degree for all prime p, i.e., $[k : k_0] = \prod_p p^{\nu_p}$ with $\nu_p < \infty$ for all p.

(b) The formal power-series field of one variable over an algebraically closed field of characteristic zero. In this note we show that the reciprocity law holds in the case (a) whereas it is not so for the case (b). Indeed we show that, for a function-field of one variable of positive genus over the field of type (b), there always exists a non-trivial (abelian) extension in which every prime divisor splits completely, i.e., an extension which can not be distinguished locally.

1. Let k be a quasi-finite field. Given a fixed generator σ of the Galois group G_k, we obtain the identification $\tilde{\sigma} : \chi(G_k) \xrightarrow{\approx} Q/Z$ given by $\chi \to \chi(\sigma)$, where $\chi(G_k)$ denotes the character group of G_k. Therefore if L is a local field with the residue field k, then we obtain Hasse invariant $\mathrm{inv}_L : B(L) \xrightarrow{\approx} Q/Z$ which is the composite of two isomorphisms $B(L) \xrightarrow{\approx} \chi(G_k) \xrightarrow{\approx} Q/Z$ where $B(L)$ is the Brauer group of L. In turn we obtain the norm residue map $(*, L) : L^* \to G_L^a$

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