

# ON ALGEBRAIC GROUPS AND DISCONTINUOUS GROUPS

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To the memory of TADASI NAKAYAMA

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## Introduction

Let  $G$  be a connected semi-simple algebraic group defined over  $\mathbf{Q}$  and let  $\Gamma$  be a discrete subgroup of  $G_{\mathbf{R}}$  (the subgroup of  $G$  consisting of points rational over  $\mathbf{R}$ ) such that  $\Gamma \backslash G_{\mathbf{R}}$  is compact. The main purpose of the present paper is to prove that for *a certain type* of group  $G$  there exists an invariant algebraic

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