ON EXTENSIONS OF TRIADS

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Dedicated to the memory of Professor TADASI NAKAYAMA

Introduction

As an extension of a result due to W. D. Barcus and J. P. Meyer [4], T. Ganea [8] has recently proved a theorem concerning the fibre of the extension $E \cup CF \to B$ of a fibre map $p : E \to B$ to the cone CF erected over the fibre F. In this paper we shall establish a generalized Ganea theorem which asserts that the homotopy type of the fibre of a canonical extension ξ' of a triad $A \xrightarrow{f} Y \xleftarrow{g} B$ (cf. [13]) is determined by those of f and g (see Theorem 3.4). This generalization yields a proof of a well-known theorem of Serre on relative fibre maps (see Corollary 3.9) and, as done by various authors (cf. [1], [10], [12]), a theorem of Blakers- Massey (see Corollary 4. 4).

Our result can be used to derive a dual EHP sequence which generalizes a conditionally exact sequence established by G. W. Whitehead [15] and Tsuchida-Ando [14]. The dual product introduced by M. Arkowitz ([2], [3]) allows us to describe the third homomorphism in that sequence.

Throughout this paper we will work in the category of spaces with base-points, generally denoted by $*$, and based maps. Homotopies are assumed to respect base-points. The closed unit interval is denoted by I. Given a path $\omega : I \to X$ in X, we denote by $\omega_{u,v}$ the path defined by $\omega_{u,v}(t) = \omega((1-t)u + tv)$, where $0 \leqq u \leqq v \leqq 1$. For paths ω, τ with $\omega(1) = \tau(0)$, the path consisting of ω followed by τ will be denoted by $\omega + \tau$, and the inverse of ω by $-\omega$. As usual, Ω and S are used, respectively, to denote the loop and suspension functors. EX and CX denote the space of paths in X emanating from the base-point and the cone over X respectively.

We are indebted to T. Ganea for sending us a preprint of [8].

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