ON QUASINORMAL SUBGROUPS II

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To the Memory of Professor TADASI NAKAYAMA

A subgroup was defined by O. Ore to be quasinormal in a group if it permuted with all subgroups of the group, and he proved [5] that such a subgroup is subnormal (= subinvariant = accessible) in a finite group. Finite groups in which all subgroups are quasinormal were classified by K. Iwasawa [3], and more recently N. Itô and J. Szép [2] and the author [1] proved that a quasinormal subgroup is an extension of a normal subgroup by a nilpotent group. Similar results were obtained by O. Kegel [4] and in [1] for subgroups which permute not necessarily with all subgroups but with those having some special property.

In this note these results are generalized to subgroups which permute with each element of a family \( \mathcal{F} \) of subgroups of the group which cover the group in a specified way. The restrictions on \( \mathcal{F} \) are slight enough to allow many different realizations in a group, including those studied in the papers indicated above. In the first section a certain normality condition is placed on the elements of \( \mathcal{F} \), and in the second section this is replaced by an arithmetic condition which enables counting arguments to be used.

All groups considered here are finite, and the following notation is used:
\( H \triangleleft G \) (\( H \lhd G \)) means that \( H \) is a normal (subnormal) subgroup of \( G \); \( \langle H, K \rangle \) is the subgroup of \( G \) generated by the subsets \( H \) and \( K \); \( H^x = x^{-1}Hx \); \( \text{Cor}_G(H) \) denotes the maximal normal subgroup of \( G \) contained in \( H \); \( N_G(H) \) is the normalizer of the subgroup \( H \) in \( G \); \( |H| \) denotes the order of the group \( H \); \( \pi_G \) denotes the subgroup of \( G \) generated by all \( \pi \)-Sylow subgroups of \( G \) for \( \pi \) in the set \( \pi \) of primes; \( \pi_G \) is written \( p_G \) when \( \pi = \{p\} \); \( G_p \) denotes a \( p \)-Sylow subgroup of \( G \); \( H^G \) denotes the normal closure of \( H \) in \( G \); and \( G^p \) is the minimal normal subgroup of \( G \) such that \( G/G^p \) is a \( p \)-group.

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