EXISTENCE OF PERFECT PICARD SETS

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Dedicated to the memory of Professor TADASI NAKAYAMA

1. Let E be a totally disconnected compact set in the z-plane and let Ω be its complement with respect to the extended z-plane. Then Ω is a domain and we can consider a single-valued meromorphic function $f(z)$ in Ω which has a transcendental singularity at each point $\zeta \in E$. Suppose that E is a null-set of the class W in the sense of Kametani [4] (= the class $N_{\mathfrak{B}}$ in the sense of Ahlfors and Beurling [1]). Then the cluster set of $f(z)$ at each transcendental singularity is the whole w-plane and hence $f(z)$ has an essential singularity at each point of E. We shall say that a value w is exceptional for $f(z)$ at an essential singularity $\zeta \in E$ if there exists a neighborhood of ζ where the function $f(z)$ does not take this value w. If each $f(z)$ has at most n exceptional values at each singularity $\zeta \in E$, we shall call E an n-Picard set using the terminology of Lehto [5] and call a 2-Picard set a Picard set simply. For any E, by Besse's theorem, there exists a single-valued regular function $g(z)$ in Ω possessing E as the set of singularities. Therefore, considering the function $\exp g(z)$ in Ω, we see that there exists no 1-Picard set. Thus we need consider n-Picard sets only for $n \geqq 2$.

For any countable E, every $f(z)$ has at most two exceptional values at each singularity $\zeta \in E$, because any neighborhood of ζ contains isolated points of E, and hence E is a Picard set. But for a non-countable E, there needs some condition in order to be an n-Picard set for some n, even if E is of logarithmic capacity zero (see Matsumoto [6]). Carleson [3] and the author [7], [8] have given sufficient conditions for sets E to be n-Picard sets for n *not smaller than* 3 and examples of perfect E by means of Cantor sets. There has remained a very interesting problem unsolved. *Is there a perfect Picard set?*

The purpose of this paper is to give Cantor sets which are Picard sets. The Schottky theorem will also play important roles as in papers [3], [7], [8].

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