ON PRIMITIVE EXTENSIONS OF RANK 3 OF SYMMETRIC GROUPS

TOSIRO TSUZUKU*

Dedicated to the memory of Professor TADASI NAKAYAMA

1. Let Ω be a finite set of arbitrary elements and let (G, Ω) be a permutation group on Ω. (This is also simply denoted by G). Two permutation groups (G, Ω) and (G, Γ) are called isomorphic if there exist an isomorphism σ of G onto H and a one to one mapping τ of Ω onto Γ such that $(g(i))^{\tau} = g^{\sigma}(i^{\tau})$ for $g \in G$ and $i \in \Omega$. For a subset Δ of Ω, those elements of G which leave each point of Δ individually fixed form a subgroup G_{Δ} of G which is called a stabilizer of Δ. A subset Γ of Ω is called an orbit of G_{Δ} if Γ is a minimal set on which each element of G induces a permutation. A permutation group (G, Ω) is called a group of rank n if G is transitive on Ω and the number of orbits of a stabilizer G_a of $a \in \Omega$, is n. A group of rank 2 is nothing but a doubly transitive group and there exist a few results on structure of groups of rank 3 (cf. H. Wielandt [6], D. G. Higman [4]).

Now we introduce the following definition:

Definition. A permutation group (G, Ω) is an extension of rank n of a permutation group (H, Γ) if (G, Ω) is a group of rank n and there exists an orbit Δ of a stabilizer G_a, $a \in \Omega$, such that G_a is faithful on Δ, i.e., only the identity element of G_a induces the identity permutation on Δ, and (G_a, Δ) is isomorphic to (H, Γ). Moreover, if (G, Ω) is primitive (or imprimitive), it is called a primitive (or imprimitive, resp.) extension of rank n.

In this note we will prove the following theorem.

THEOREM. *Let S_n be the symmetric group of degree n.*
If S_n has a primitive extension of rank 3, *then* $n = 1, 2, 3, 5,$ *or* 7.

2. We use the following notations:

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