TRANSITIVE EXTENSIONS OF A CLASS OF DOUBLY TRANSITIVE GROUPS

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To the memory of Professor TADASI NAKAYAMA

1. When a permutation group G on a set Ω is given, a transitive extension G_1 of G is defined to be a transitive permutation group on the set Γ which is a union of Ω and a new point ∞ such that the stabilizer of ∞ in G_1 is isomorphic to G as a permutation group on Ω. The purpose of this paper is to prove that many known simple groups which can be represented as doubly transitive groups admit no transitive extension. Precise statement is found in Theorem 2. For example, the simple groups discovered by Ree [5] do not admit transitive extensions. Theorem 2 includes also a recent result of D. R. Hughes [3] which states that the unitary group $U_3(q)$ $q>2$ does not admit a transitive extension. As an application we prove a recent theorem of H. Nagao [4], which generalizes a theorem of Wielandt on the non-existence of 8-transitive permutation groups not containing the alternating groups under Schreier's conjecture.

2. We will introduce notation which will be used throughout this paper.

Let G be a doubly transitive group on Ω and let H be the stabilizer of a point a of Ω. Suppose that a conjugate class C of G consisting of elements of order 2 is given. Then there is an element s of C such that we have a decomposition of G into a union of two double cosets:

$$G = H \cup HsH \text{ where } s \in C.$$

We define

$$D = H \cap H^s,$$

and remark that s normalizes D.

For a transitive extension, a theorem of E. Witt is fundamental (Witt [10]), which will be stated here as Lemma 1.

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