ON THE EXPLICITE DEFINING RELATIONS OF ABELIAN SCHEMES OF LEVEL THREE

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Dedicated to the memory of Professor TADASI NAKAYAMA

It is known classically that abelian varieties of dimension one over the field of complex numbers may be expressed by non-singular *Hesse's canonical cubic plane curves*, $X_0^3 + X_1^3 + X_{-1}^3 - 6\gamma X_0 X_1 X_{-1} = 0$. The purpose of the present paper is to generalize this idea to higher dimensional case.

Let $\mathbf{Z}(3)$ be the residue group of the additive group $\mathbf{Z}$ of integers modulo $3\mathbf{Z}$ and $\mathbf{Z}(3)^r$ be the r-times direct sum of $\mathbf{Z}(3)$. We mean by $\mathbf{Z}(3)^{+r}$ the subset of $\mathbf{Z}(3)^r$ consisting of all the elements $(a_1^+, \dots, a_r^+)$ such that $a_i^+ = 0$ or 1 $(1 \le i \le r)$. Then, roughly speaking, our result may be expressed as follows: a generic abelian variety with a positive divisor U such that $l(U) = 1$[1] is defined by relations of the following type

$$(*)\qquad \Delta_2 Y_{a+b} Y_{-a+b} Y_b - \sum_{c \in \mathbf{Z}(3)^r} \gamma_{a,c} Y_{c+b}^3 = 0$$

$$(**)\qquad \Delta_1 Y_{a+b} Y_{-a+b} - \sum_{c^+ \in \mathbf{Z}(3)^{+r}} \beta_{a,c^+} Y_{c^+ + b} Y_{-c^+ + b} = 0, \qquad (a, b \in \mathbf{Z}(3)^r).$$

§ 1. Formal theta functions of level n and the scheme $A(r, n)$ associated with them

1.1. We mean by $\mathbf{Z}$ and $\mathbf{Q}$ the ring of intergers and the field of rational numbers. We mean by $\mathbf{Z}^r$ the r-times direct sum of the $\mathbf{Z}$-module $\mathbf{Z}$ and by $\mathbf{Q}^r$ the r-times direct sum of the $\mathbf{Q}$-module $\mathbf{Q}$. Let $\{W(i;\alpha),\ W(j,l;\beta) \mid 1 \le i, j, l \le r;\ \alpha, \beta \in \mathbf{Q}\}$ be a system of indeterminates on which rational numbers operate such that $W(i;\alpha)^\gamma = W(i;\alpha\gamma)$, $W(j,l;\beta)^\gamma = W(j,l;\beta\gamma)$. We denote by I the ideal in the polynomial ring $\mathbf{Z}[\{W(i;\alpha),\ W(f,l;\beta)\}]$ generated by

$$W(i;0) - 1,\ W(j,l;0) - 1,\ W(i;n\alpha) - \overbrace{W(i,\alpha) \cdots W(i;\alpha)}^{n},$$
$$W(j,l;n\beta) - \overbrace{W(j,l,\beta) \cdots W(j,l;\beta)}^{n}$$

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[1] $l(U)$ means the rank of the module of the multiples of $-U$.