AN EXISTENCE THEOREM IN POTENTIAL THEORY

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Dedicated to the memory of Professor TADASI NAKAYAMA

1. Concerning a positive lower semicontinuous kernel G on a locally compact Hausdorff space X the following existence theorem was obtained in [3].

THEOREM A. *Assume that the adjoint kernel* $\check{G}$ *satisfies the continuity principle. Then for any separable compact subset* K *of* X *and any positive upper semicontinuous function* $u(x)$ *on* K, *there exists a positive measure* μ, *supported by* K, *such that*

$$G\mu(x) \geq u(x) \qquad G\text{-}p.p.p. \text{ on } K,$$
$$G\mu(x) \leq u(x) \qquad \text{on } S\mu, \text{ the support of } \mu.$$

Nakai [4] proved the theorem without assuming the separablility of K. Using Kakutani's fixed-point theorem he simplified a part of the proof. But he needed prudent considerations on topology in order to avoid the separability. In this paper we shall give a simpler proof of the theorem without assuming the separability. We shall deal with a slightly more general kernel and use Glicksberg-Fan's fixed-point theorem.

2. A lower semicontinuous function $G(x, y)$ on $X \times X$ with $0 \leq G(x, y) \leq +\infty$ is called a non-negative l.s.c. kernel on X. The kernel $\check{G}$, defined by $\check{G}(x, y) = G(y, x)$, is called the adjoint kernel of G. The potential $G\mu(x)$ of a positive measure μ is defined by $G\mu(x) = \int G(x, y)\, d\mu(y)$. The adjoint potential $\check{G}\mu(x)$ is similarly defined. The adjoint kernel $\check{G}$ is said to satisfy the continiuity principle when finite continuous is every adjoint potential $\check{G}\mu$ of a positive measure μ with compact support which is finite continuous as a function on $S\mu$.

3. We shall prove

Received April 12, 1965.