NIL SEMI-GROUPS OF RINGS WITH A POLYNOMIAL IDENTITY

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In the memory of Professor TADASI NAKAYAMA

The basic properties of associative rings R satisfying a polynomial identity $p[x_1, \ldots, x_n] = 0$ were obtained under the assumptions that the ring was an algebra [e.g., [4] Ch. X], or with rather strong restrictions on the ring of operators ([1]). But it is desirable to have these properties for arbitrary rings, and the present paper is the first of an attempt in this direction. The problem is almost trivial for prime or semi-prime rings but quite difficult in arbitrary rings. The known proofs for algebras have to be modified and in some cases new proofs have to be obtained as the existing proofs fail to exploit the known structure. In the present paper we extend the results of [1] on the nil subalgebras of a ring with an identity for arbitrary multiplicative nil semi-groups of the ring and for arbitrary rings.

Finally, we extend our results to rings with a pivotal monomial and as a consequence we show that the nil multiplicative semigroups of a simple ring of bounded index are nilpotent.

1. Notations. Let Ω be a set of linear mappings of a ring R into a ring T, i.e., given a mapping $\Omega \times R \to T$, denoted by $w.r.$ and satisfying

$$(1.1) \qquad \begin{array}{l} w(rs) = (wr)s = r(ws) \\ w(r+s) = wr + ws \end{array}, \quad w \in \Omega;\ r,\ s \in R.$$

Let $x_1, x_2, \ldots,$ be an infinite set of indeterminates. Let $\tilde{\Omega}[x]$ be the free ring generated by the $\{x_i\}$ and the symbols of Ω, and among the elements of $\tilde{\Omega}[x]$, we restrict ourselves to the set $\Omega[x]$ of all polynomials $p[x] = \sum w_{(i)} x_{i_1} \cdots x_{i_n}$ which are finite sums of different monomials $x_{i_1} \cdots x_{i_n}$ preceded by an element $w_{(i)}$ of the set Ω.

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