RADON-NIKODYM DENSITIES BETWEEN HARMONIC MEASURES ON THE IDEAL BOUNDARY OF AN OPEN RIEMANN SURFACE

MITSURU NAKAI

Dedicated to the memory of Professor TADASI NAKAYAMA

1. Resolutive compactification and harmonic measures. Let R be an open Riemann surface. A compact Hausdorff space R^* containing R as its dense subspace is called a *compactification* of R and the compact set $\Delta = R^* - R$ is called an *ideal boundary* of R. Hereafter we always assume that R does not belong to the class O_G. Given a real-valued function f on Δ, we denote by $\overline{\varphi}_f^{R,R^*}$ (resp. $\underline{\varphi}_f^{R,R^*}$) the totality of lower bounded superharmonic (resp. upper bounded subharmonic) functions s on R satisfying

$$\liminf_{R \ni p \to p^*} s(p) \geq f(p^*) \qquad (\text{resp. } \limsup_{R \ni p \to p^*} s(p) \leq f(p^*))$$

for any point p^* in Δ. If these two families are not empty, then

$$\overline{H}_f^{R,R^*}(p) = \inf\,(s(p)\,;\ s \in \overline{\varphi}_f^{R,R^*}) \text{ and } \underline{H}_f^{R,R^*}(p) = \sup\,(s(p)\,;\ s \in \underline{\varphi}_f^{R,R^*})$$

are harmonic functions on R and $\overline{H}_f^{R,R^*} \geq \underline{H}_f^{R,R^*}$ on R. If these two functions coincide with each other on R, then we denote by H_f^{R,R^*} this common function and call f *resolutive* with respect to R^* (or Δ). We denote by $C(\Delta)$ the totality of bounded real valued continuous functions on Δ. If any function in $C(\Delta)$ is resolutive with respect to Δ, then following Constantinescu and Cornea [1] we say that R^* is a *resolutive compactification* of R. Important examples of resolutive compactifications are Wiener's, Martin's Royden's, Kuramochi's and Kerékjártó-Stoilow's compactifications (see [1]). Hereafter we always consider the resolutive compactification R^* of R.

Fix a point p in R. It is easy to see that $f \to H_f^{R,R^*}(p)$ is a positive linear functional on $C(\Delta)$ and so by Riesz-Markoff-Kakutani's theorem, there exists a positive regular Borel measure μ_p on Δ such that

Received February 24, 1965.