IMBEDDING A REGULAR RING IN A REGULAR RING WITH IDENTITY

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Dedicated to the memory of Professor TADASI NAKAYAMA

In [1] L. Fuchs and I. Halperin have proved that a regular ring R is isomorphic to a two-sided ideal of a regular ring with identity. ([1] Theorem 1). Their methed is to imbed the regular ring R in the ring of all pairs (a, ρ) with $a \in R$ and ρ from a suitable commutative regular ring S with identity such that R is an algebra over S. Thus S may be seen as the ring of $R-R$ endomorphisms of the additive group of R. The following question is naturally raised: Is it true that the ring of all $R-R$ endomorphisms of a rugular ring is a commutative regular ring? The main purpose of this paper is to answer this question affirmatively. (Theorem 1). After established this theorem we can follow the method in [1] to solve the problem in the title.

1. Endorphisms of R^+.

Let R^+ be the additive group of a given ring R with R as left and right operator domains, and let $\tilde{R}$ be the ring of all endomorphisms of R^+, that is the ring of all $R-R$ endomorphisms of the additive group R. $\tilde{R}$ has the identity $\bar{1}$ which is the identity mapping of R^+. Also let us denote by $\bar{0}$, $\bar{n}$ and $\bar{c}$ respectively the zero endomorphism, $\bar{n}$: $a \to na$, where a is an element in R and n is an integer, $\bar{c}$: $a \to ac$, where c is an element in the center C of R.

LEMMA 1. *If R has the identity* 1, *then $\tilde{R}$ is isomorphic to the center C of R.*

Proof. Let ρ be an element of $\tilde{R}$. Then for any element a in R we have $a\rho = (a1)\rho = a(1\rho)$ and $a\rho = (1a)\rho = (1\rho)a$. Thus $c = 1\rho$ is in the center C of R and $a\rho = ac = ca$. Conversely let c be an element in C, then $\bar{c}$: $a \to ac$ is an endomorphism of R^+. $\rho \to 1\rho$ sets up a ring isomorphism between $\tilde{R}$ and C.

LEMMA 2. *If $R^2 = R$, then $\tilde{R}$ is commutative.*

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