SOME RESULTS IN THE FOURIER ANALYSIS

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To TADASI NAKAYAMA in memorial

There are many uses of Fourier analysis in the analytic number theory. In this paper we shall derive two fundamental theorems using Cramer's method (Mathematical methods of statistics, 1946). Let E, E^* be unit cubes in the whole n-dimensional Euclidean space X such that

$$E = \{(u_1 \cdots u_n) : 0 \leqq u_1 \leqq 1, \ldots, 0 \leqq u_n \leqq 1\}$$

$$E^* = \left\{(u_1 \cdots u_n) : x_1 - \frac{1}{2} \leqq u_1 \leqq x_1 + \frac{1}{2}, \ldots, x_n - \frac{1}{2} \leqq u_n \leqq x_n + \frac{1}{2}\right\}$$

We define $F(u)$ as follows

$$F(u) = 0\ (u < x - t),\ \frac{1}{2}\ (u = x - t),\ 1\ (x - t < u < x + t),$$

$$\frac{1}{2}\ (u = x + t),\ 0\ (x + t < u),$$

for fixed x and $t > 0$.

LEMMA 1. *For fixed x and $t\left(0 < t < \frac{1}{2}\right)$ the function*

$$\sum_{m=-k}^{k} \frac{\text{sim}\ 2\pi m(x + t - u)}{2\pi m} - \sum_{m=-k}^{k} \frac{\sin 2\pi m(x - t - u)}{2\pi m} \tag{1}$$

is boundedly convergent to $F(u)$ as $k \to \infty$, where $x - \frac{1}{2} \leqq u \leqq x + \frac{1}{2}$.

Proof. Since (1) is equal to

$$2t + 2\int_0^{x+t-u} (\cos 2\pi z + \cdots + \cos 2\pi kz)\, dz - 2\int_0^{x-t-u} (\cos 2\pi z + \cdots + 2\pi kz)\, dz$$

$$= 2t + 2\int_{x-t-u}^{x+t-u} \frac{\sin\left(k + \frac{1}{2}\right) 2\pi z - \sin \frac{1}{2} 2\pi z}{2 \sin \frac{1}{2} 2\pi z}\, dz = \int_{x-t-u}^{x+t-u} \frac{\sin(2k+1)\pi z}{\sin \pi z}\, dz,$$

the lemma is obtained by proving that

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