CHARACTERIZATION OF FINITE DEDEKIND GROUPS

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If $U = U(0)$, $U(i)$ is a maximal subgroup of $U(i+1)$ and $U(n) = V$, then we term the $U(i)$ a densest chain connecting U and V; and n is the length of this chain. The subgroup U of V will furthermore be termed an n-uniserial subgroup of V if there exists one and only one densest chain connecting U and V and if its length is n. The principal aim of this note is to give characterizations of dedekind groups [= groups all of whose subgroups are normal] in terms of the normality of uniserial subgroups. We quote one of our results:

The finite group G is a dedekind group if, and only if, all uniserial subgroups of G are normal and normal subgroups of G normalize their 2-uniserial subgroups [Corollary 2.14].

Our principal results [Theorems 2.10 and 3.3] are a little more differentiated. The method employed consists in an exact determination of all finite groups that are not dedekind groups, though all their proper epimorphic images are dedekind groups and which in addition meet certain definite requirements [Proposition 2.1].

Notations

$\mathbf{z}G$ = center of G.

$\mathbf{c}U = \mathbf{c}_G U$ = centralizer of U in G.

$x \circ y = x^{-1}y^{-1}xy$

$x^y = x(x \circ y) = y^{-1}xy$

$U \circ V$ = subgroup generated by all the $u \circ v$ with u in U and v in V.

$G' = G \circ G$ = commutator subgroup of G.

Factor of G = epimorphic image of subgroup of G.

$\{\cdots\}$ = subgroup generated by the enclosed set.

All groups considered are *finite!*

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