A CHARACTERIZATION OF QF-3 RINGS

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To the memory of TADASI NAKAYAMA

A left QF-3 ring R is one in which ${}_RR$, *the ring considered as a left module* over itself, can be embedded in a projective injective left R module $Q({}_RR)$. QF-3 rings were introduced by Thrall [14] and have been studied and characterized by a number of authors [5, 8, 9, 12, 13, 15] usually restricted to the case of algebras over a field. In such a case, the concept of left QF-3 and right QF-3 coincide.

The study of QF-3 rings and algebras and many other such classes of rings had its origin in the now classic papers of Nakayama [10, 11]. He was an outstanding pioneer in algebra for many years, and we acknowledge our great debt to him and to his many excellent papers.

In this note we shall restrict consideration to the class of rings with minimum condition in left ideals and within the class we shall give a new characterization of left QF-3 rings. Actually, our characterization holds for a more general class of rings. See the remarks at the end of the note.

Incidently, for rings with minimum codition both on left and on right ideals, we do not know if the concepts of left QF-3 and right QF-3 coincide as they do for the case of finite dimensional algebras over fields.

Throughout the paper R will represent a ring with minimum condition on left ideals and all modules are left R modules. When R appears as a module, it is as a left module over itself. Hom(X, Y) will always mean Hom$_R(X, Y)$ for R-modules X, Y.

Our characterization of QF-3 rings is given in terms of certain classes of modules. A module M is called ***torsionless*** if for each $m \in M$, $m \neq 0$, there exists $f \in$ Hom(M, R) such that $f(m) \neq 0$. M is torsionless if there are enough homomorphisms of M to R to distinguish points of M from 0. The concept

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