## ELEVEN NONEQUIVALENT CONDITIONS ON A COMMUTATIVE RING

## ROBERT W. GILMER, JR.

1. Introduction. We consider in this paper eleven conditions on a commutative ring R. The first of these is that R contains an identity. It is well known that each of the other properties is a consequence of the first condition. This paper considers other relations which exist between these properties. A complete diagram of all simple implications which exist between the eleven properties, together with proof of these implications, is given in section 3. Examples illustrating simple implications which do not hold are presented in section 4. The notation and terminology is that of [10] with one exception:  $\subseteq$  denotes containment and  $\subset$  denotes proper containment. All rings considered will be assumed to be commutative and nonzero.

The eleven conditions considered on R, a commutative ring are:

- A: R contains an identity.
- B: R is generated by idempotent elements.
- C: If A is a nonzero ideal of R such that  $\sqrt{A} \neq R$ , then R/A has an identity.
- D: If  $x \in R$ , there exists  $y \in R$  such that x = xy.
- E: If A is a proper ideal of R,  $\sqrt{A} \neq R$ .
- F: R is idempotent.
- G: Maximal ideals of R are prime.
- H: If P is a nonzero prime ideal of R, R/P contains an identity.
- J: An ideal A such that  $\sqrt{A}$  is maximal is primary.
- K: Each proper ideal of R is contained in a maximal ideal.
- L: If A and B are comaximal proper ideals of R, then  $AB = A \cap B$ .

It is well-known that properties B-L follow from A. Further, each of these properties is preserved under homomorphisms. Rings satisfying E have arisen naturally in [2], [3], and [4], and according to the terminology used

Received November 17, 1964.