# A REMARK ON THE INTERSECTION OF TWO LOGICS

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The intuitionistic logic **LJ** and Curry's **LD** (cf. [1], [2]) are logics stronger than Johansson's minimal logic **LM** (cf. [3]) by the axiom schemes $\curlywedge \rightarrow x$ and $y \vee (y \rightarrow \curlywedge)$, respectively. However, **LM** can not be taken literally as the intersection of these two logics **LJ** and **LD**, which is stronger than **LM** by the axiom scheme $(\curlywedge \rightarrow x) \vee y \vee (y \rightarrow \curlywedge)$. In pointing out this situation, Prof. K. Ono suggested me to investigate the general feature of the intersection of any pair of logics. In this paper, I will show that the same situation occurs in general. I wish to express my thanks to Prof. K. Ono for his kind guidance.

Let **A** be a logic having logical constants, *implication* ($\rightarrow$) and *disjunction* ($\vee$) (and *universal quantification* ( ) for predicate logics), together with all such inference rules with respect them that are admitted in the intuitionistic logic (cf. [5], p. 81). For any logic **L**, let us denote by $\Pi_L$ the class of all provable propositions in **L**.

THEOREM. *Let* **B**, **C**, *and* **D** *be the logics formed from* **A** *by adjoining the axiom schemes*

(1) $(u_1)\cdots(u_p) f(x_1, \ldots, x_s), \qquad (p = 0, 1, 2, \ldots),$

(2) $(v_1)\cdots(v_q) g(y_1, \ldots, y_t), \qquad (q = 0, 1, 2, \ldots),$

(3) $(u_1)\cdots(u_p) f(x_1, \ldots, x_s) \vee (v_1)\cdots(v_q) g(y_1, \ldots, y_t), \qquad (p, q = 0, 1, 2, \ldots),$

*respectively; where* $u_i$*'s and* $v_j$*'s are object variables* ($p = q = 0$ *for proposition logics*), $(u_1)\cdots(u_p) f(x_1, \ldots, x_s)$ *and* $(v_1)\cdots(v_q) g(y_1, \ldots, y_t)$ *are expressible in* **A**, $x_i$*'s and* $y_j$*'s are metalogical variables for propositions, predicates, or relations, and* $s \leq t$. *Then,*

I. $$\Pi_D = \Pi_B \cap \Pi_C.$$

II. **B** *and* **C** *formed from* **D** *by adjoining the axiom schemes*

$(4)_\mu$ $(w_1)\cdots(w_r)(g(y_1, \ldots, y_t) \rightarrow f(y_{\mu(1)}, \ldots, y_{\mu(s)})), \qquad (r = 0, 1, 2, \ldots),$

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