A NEW DEFINITION OF THE n-DIMENSIONAL QUASICONFORMAL MAPPINGS

PETRU CARAMAN

Introduction

In this note we shall extend, for arbitrary n, Pesin's [11] bidimensional definition for quasiconformal mappings and establish its equivalence with Gehring's [7] and Väisälä's [15] definitions.

The four Väisälä's [15] definitions are the following:

1° A homeomorphism $\bar{x}=f(x)$ of a domain $D\subset R^n$ is called *K-quasiconformal* $(1\leqq K<\infty)$, if $\delta(x)$ is uniformly bounded in D and $\delta(x)\leqq K$ a.e.[1] in D, where, for each r, $0<r<d\ (x, frD)$, we put (according to Väisälä [15]):

$$L(x,r)=\max_{|x'-x|=r}|f(x')-f(x)|,\qquad l(x,r)=\min_{|x'-x|=r}|f(x')-f(x)|,$$

$$T(x,r)=m\{f[B(x,r)]\},$$

$$\bar{\delta}(x)=\overline{\lim_{r\to 0}}\frac{A_nL(x,r)^n}{T(x,r)},\qquad \underline{\delta}(x)=\underline{\lim}_{r\to 0}\frac{T(x,r)}{A_nl(x,r)^n},$$

$$\delta(x)=\max[\bar{\delta}(x),\ \underline{\delta}(x)],\qquad \delta_L(x)=\overline{\lim_{r\to 0}}\frac{L(x,r)}{l(x,r)}$$

and A_nr^n is the volume of the n-dimensional ball $B(x,r)$ wtih the centre x and the radius r.

2° A homeomorphism $\bar{x}=f(x)$ *of a domain* $D\subset R^n$ is called *K-quasiconformal* $(1\leqq K<\infty)$ if

$$\frac{1}{K}M(\Gamma)\leqq M(\Gamma^*)\leqq KM(\Gamma)$$

for each curve family $\Gamma\subset D$, where $M(\Gamma)$ is the module of Γ and Γ^* is its image.

We recall (see Fuglede [5]) that

$$M(\Gamma)=\inf_{\rho\in F(\Gamma)}\int_{R^n}\rho^n d\tau,$$

Received July 23, 1965.

[1] a.e. = almost everywhere.